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Scaling on the mobility threshold in ultra-thin caesium films

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Abstract. The dependence of resistance upon thickness and temperature has been calculated for a thin continuous metal film. For a thickness near that of a monolayer, quantum corrections to conductivity are of the order of the classical conductivity itself, making a film behave like a semiconductor. Computed dependences have a striking resemblance to experimental data obtained on quench-condensed alkali metal films.

1. Introduction

It is widely agreed, since the early work of Abrahams *et al* (1979), that at absolute zero any two-dimensional system must be an insulator. This means that for every metal film there is a certain temperature below which the film has to show semiconductor properties and rapidly become an insulator upon further cooling. However, it is rather difficult to get a strong localization state for a continuous metal film because of the weakness of the quantum corrections to conductivity in the two-dimensional case and the metal regime. It is easy to estimate that, at reasonable temperatures, strong localization may occur only for a film with average thickness t of the order of one monatomic layer a, since the conductance of such a film with mean free path l of the order of a monolayer $l \simeq t \simeq a$ is about e^2/h. Normally quench-condensed metal films do not conduct when they are less than two or three monolayers thick, so the metal–insulator transition is taking place in a system of separate metal islands connected with quantum tunnelling, rather than in a continuous film.

Probably the thinnest continuous metal films known at present are quench-condensed alkali metal films made under the method proposed and developed by Shalnikov (Astrakharchik and Shalnikov 1977). They start to conduct at a thickness of about 0.7 of a monatomic layer. Conduction becomes metallic for films of one monolayer with a resistance of 80 kΩ. It is difficult to tell which feature of Shalnikov's method is responsible for such a low conduction thickness. It could be due to the special shape of the evaporation source prevented penumbra. It may be due to a low temperature of metal evaporation and a good thermal contact of a substrate with a helium bath. It could also be simply a property of alkali metals with a single conduction electron to make metal links between separate atoms at a very early stage of film growth. Whatever the reason, the films were very homogeneous and, as will be shown from further consideration, could be treated as continuous even at a thickness below monatomic in a strong exponential localization regime.

Of course, the terms 'continuous' and 'homogeneous' should be very carefully used when referring to a film of average thickness of a monolayer or less, and made of randomly distributed atoms. It means only that the classical resistance of the film (or rather a

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conductive metal network) is of the same order as that of a homogeneous film with the same thickness, except for a narrow region just near the percolation threshold ($\simeq 0.5$ monolayer).

2. Conductivity of a thin metal film

We will now try to estimate quantum corrections to conductivity for an ultra-thin metal film, and to show that there is a rather large thickness region, where the film, being weakly localized at distances of the order of the thickness, shows strong localization behaviour and high resistance when measured at long distances. For a metal with one conduction electron the classical film conductance is

$$g_c \equiv (h/e^2)R^{-1} = (\tfrac{8}{3}\pi)^{1/3}(tl/a^2) \tag{1}$$

where the mean free path l depends on the thickness t by the well known Fuchs formula

$$l = l_\infty\left\{1 - \frac{3l_\infty}{2t}\int_1^\infty (x^{-3} - x^{-5})\left[1 - \exp\left(-\frac{tx}{l_\infty}\right)\right]dx\right\} \tag{2}$$

where l_∞ is a mean free path in the bulk metal. To take into account quantum corrections we have to integrate the Gell-Mann–Low equation $\beta(g) = d\ln g/d\ln L$, defining the change of conductance g with scaling of size L, from a length of the order of the film thickness to a sample size at absolute zero or to a phase-breaking length, that plays the role of the sample size at non-zero temperatures. For caesium films phase-breaking times have been thoroughly studied (Astrakharchik 1988, 1990). At liquid helium temperatures they were found to be determined mainly by inelastic scattering. The characteristic frequency of the scattering τ_i^{-1} for all the studied films ($1 < g < 250$) was linearly dependent on temperature $h\tau_i^{-1} \simeq kT$ with a coefficient of the order of unity, independent of the film thickness and resistance. This gives the value $L_i \simeq \sqrt{lv_F h/kT}$ for the phase-breaking length, where v_F is the Fermi velocity.

When classical conductance g_c is found by substituting (2) into (1), the conductance with quantum corrections g_q could be found as the upper limit of integration of the Gell-Mann–Low function

$$\int_{g_c}^{g_q} \frac{d\ln g}{\beta(g)} = \int_l^{L_i} d\ln l = \ln\sqrt{lv_F h/kT} - \ln t. \tag{3}$$

3. The Gell-Mann–Low function

At this point we must choose something definite as the Gell-Mann–Low function. It could be understood from first principles (Abrahams *et al* 1979) that the function has asymptotes $\beta(g \to 0) \simeq \ln g$ for the strong localization and $\beta(g \to \infty) \simeq -1/g$ for the weak one. The simplest function providing both these asymptotes is

$$\beta(g) = \ln(g+1) - \ln g = -\ln(1 + g^{-1}). \tag{4}$$

An additional argument for this choice is that the function joins the asymptotes together at a very short interval and it is known from experiments on thin films that the transition from strong to weak localization happens rather quickly, with no intermediate regime.

4. Numerical results and comparison with experiment

The result of the calculation of conductivity upon thickness is shown in figures 1 and 2. First, the classic Fuchs dependence (full curve) was fitted to experimental data for a caesium film at temperature 4.2 K (circles) using the only adjustable parameter l_∞. Next, (3) was integrated using (4) to obtain the conductivity with quantum corrections (broken curve). At large thicknesses (figure 1) the latter is simply shifted down by a relatively small value $\ln(L_i/t) \simeq 5$. However, when the thickness approaches that of a monolayer or less, and the classic conductivity decreases below that value, the quantum corrections drastically change the resulting conductivity (figure 2). At small distances, of the order of the film thickness, electrons remain delocalized and have metal wavefunctions, as long as the full curve is still above the level $\ln g = 0$. However, at distances larger than L_i, localization provides a rather large resistance (broken curve). It can be seen that this simplistic model gives results very close to the experimental data.

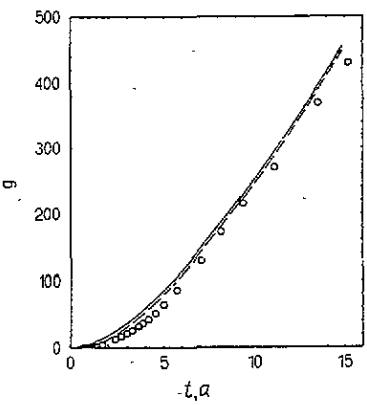

Figure 1. The dependence of conductance g in dimensionless units of (e^2/h) upon thickness in monolayers. Circles: a quench-condensed Cs film at temperature 4.2 K. Full curve: classical Fuchs conductance with $l_\infty = 25a$. Broken curve: conductance with quantum corrections.

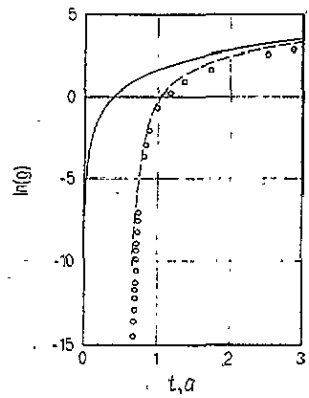

Figure 2. The same as figure 1, but with logarithmic scale of conductance.

The same equations (3) and (4) could also be used to determine the dependence of conductivity upon resistance at a fixed thickness (figure 3). The temperature, in such a case, only changes the effective sample size L_i in (3). However, the resulting curves show semiconducting behaviour, again rather close to that observed in the experiment. It is worthwhile mentioning that the only adjustable parameter for all the curves shown in figures 1, 2 and 3 is the mean free path in the infinitely thick film $l_\infty \simeq 25a$, obtained from the slope $g(t)$ for thick films (figure 1).

The phenomena considered must always be present at low-temperature metal-to-insulator transitions in any film, or any other two-dimensional structure, whatever the reason for the change in classical conductivity. If, for example, one attempts to study island film percolation dependences, then the quantum corrections must first be subtracted from the film resistance. As can be seen from the consideration above, for the quench-condensed caesium film such a subtraction provides nothing but the continuous film dependence of resistance upon thickness, even at a thickness as small as 0.8 monolayer.

Having shown that the scaling approach gives a pattern rather close to experiment, we can try to determine the Gell-Mann–Low function from the experimental data. The most

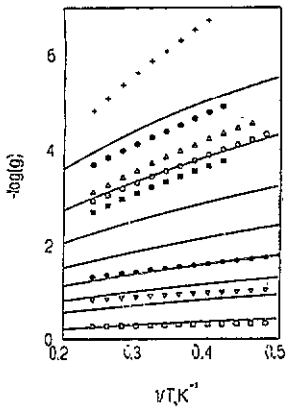

Figure 3. 'Semiconducting' temperature; decimal logarithm of a film resistance as a function of inverse temperature. Different symbols stand for different Cs films with thicknesses from 0.8–1 monolayer. Full cures are results of integrating (3) for a set of thicknesses.

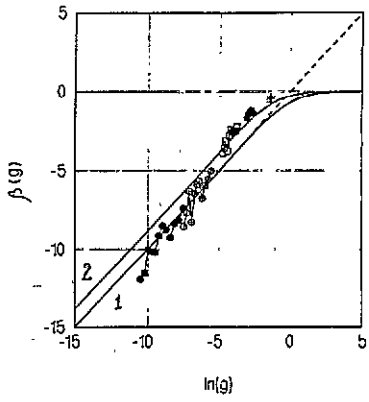

Figure 4. Line 1: the Gell-Mann–Low function as (4). Line 2: the same with conductance measured in the two-dimensional units $e^2/2\pi^2\hbar$. Experimental points: $(\mathrm{d}\ln g)/(\mathrm{d}\ln L_i)$ for Cs films with $\tau_i^{-1} \approx kT$.

direct way is to calculate it from the temperature dependences of conductivity. The same method, with rather similar results, was used by Zavaritskaya *et al* (1985, 1987), where the inelastic scattering was not measured but just supposed to be of the order of the temperature.

If $h\tau_i^{-1} \simeq kT$ then $\beta(g) = \mathrm{d}\ln g/\mathrm{d}\ln L = 2(\mathrm{d}g/g)(T/\mathrm{d}t)$. The dependence for caesium films with different thicknesses is plotted in figure 4, where the curves for different films construct the single dependence close to (4). The accuracy of the determination could be improved further, by approximating the experimental inelastic scattering dependence as $\tau_i^{-1} \simeq T^P$. The result of division of the data found experimentally, $P \simeq 1.2$, is very close to curve 2 in figure 4. The curve corresponds to the function $\beta(g)$ as (4) but with g counted in two-dimensional units $2\pi^2\hbar/e^2$, as would be expected.

5. Summary

In conclusion, I wish to emphasize that the same regime, when conductance just seems to be hopping and activational, while the temperature actually still affects nothing but the effective sample size, must be effective in any two-dimensional system close to the metal–insulator transition at sufficiently low temperatures.

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